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## DESIGN AND PERFORMANCE OF THROTTLE-TYPE FUEL CONTROLS FOR ENGINE DYNAMIC STUDIES

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## SUMMARY

The results of an analytical and experimental investigation of the steady-state and dynamic characteristics of three types of throttle-controlled fuel systems are presented. The three systems are (1) a throttle with regulated upstream pressure, (2) a throttle plus bypass differential-pressure regulator, and (3) a throttle plus reducing-valve differential-pressure regulator with regulated supply pressure. Performance and stability criteria based on a linearized analysis are presented for the three systems.

Experimental data for the three systems show (1) the effect of output pressure on the controlled flow, (2) the response of output flow to step changes in throttle area, and (3) the response of output flow to a sinusoidal variation in throttle area.

The experimental results show that linearized analysis provides an adequate description of the dynamic response. Of the three systems tested, the reducing-valve system showed the closest agreement with linearized analysis and responded to the highest frequencies. The response of this system was adequate to 150 cycles per second and usable to 300 cycles per second.

## INTRODUCTION

Experimental research on automatic control of gas-turbine engines requires methods of control of engine fuel flow that provide essentially linear flow response to input signals over a frequency band extending from zero to approximately 50 cycles per second. This report presents the results of an investigation into the basic characteristics of three valve-controlled systems that may be utilized for this purpose. The three systems are (1) a throttle (valve) with a regulated (constant) upstream pressure, (2) a throttle with a regulated (constant) pressure differential (bypass-type regulator), and (3) a throttle with a regulated pressure differential (reducing-valve regulator).

These three systems of flow control are widely known and have long been applied in other fields. However, in the present application for gas-turbine engines, the requirement of an adequate flow response at a 50-cycle-per-second input signal makes it necessary to consider dynamic effects not often encountered in previous applications. An analysis was therefore made of the factors affecting the response of the three systems at such relatively high input-signal frequencies. This analysis is an extension of the analysis made in reference 1.

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This report presents an analysis of the steady-state and dynamic characteristics of the three systems. The principal figure of merit used in the analysis is related to the effect of output pressure on the controlled flow. This figure of merit, which is expressed as the ratio of the change in output pressure to the resulting change in output flow, is called the output-flow impedance.

Systems that exhibit the highest values of output-flow impedance yield the most linear response of flow to throttle area. Expressions are derived that give the output-flow impedance in terms of the physical dimensions of the systems. The analysis also yields a stability criterion and the characteristic time constant for overdamped systems.

Measurements of the output-flow impedance and of the transient and frequency response of the three systems are presented along with the analytically determined frequency responses. The investigation was conducted at the NACA Lewis laboratory.

## SYMBOLS

The following symbols are used in this report:

A area, sq in.

B fluid bulk modulus, lb/sq in.

D dimensional constant in orifice equation, sq in./sec$\sqrt{\text{lb}}$

K spring constant, lb/in.

M mass, (lb)(sec$^2$)/in.

P pressure, lb/sq in.

$P_t$ pressure difference across throttle, lb/sq in.

Q flow, cu in./sec

| | |
|---|---|
| $V$ | conduit volume, cu in. |
| $w$ | width of valve port, in. |
| $x$ | axial position of piston and valve element, in. |
| $\delta$ | damping ratio |
| $\tau$ | time constant, sec |



Subscripts:

| | |
|---|---|
| C | combustion chamber |
| c | compressibility |
| d | drain |
| G | pump |
| o | output |
| p | piston |
| r | regulated |
| s | supply |
| t | throttle |
| v | control valve |



## DESCRIPTION AND ANALYSIS OF SYSTEMS

In this section three systems will be discussed: (1) a throttle with a regulated upstream pressure, (2) a throttle plus relief-valve differential-pressure regulator, and (3) a throttle plus reducing-valve differential-pressure regulator with regulated supply pressure. The three systems will be examined with respect to their ability to maintain their output flow as a function only of the throttle setting (independent of variations in the output pressure). This constant-flow characteristic corresponds in electrical analogy to a generator with high internal resistance or, for the a-c case, high internal impedance. Because the internal impedance is most often measured by varying the external loading, it has become common practice to call it the output impedance. The output impedance for the flow-control systems is herein defined as the ratio of the change in output pressure to the change in output flow $\Delta P_o/\Delta Q_o$. The frequency band over which the output impedance can be maintained at its zero-frequency value is indicated by the frequency band over which the response of output flow to throttle variations can be maintained.

## Throttle System

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Description of system. - A general arrangement of a throttle-control system with a regulated supply pressure feeding an engine fuel system is shown in figure 1(a). In the figure a common method of obtaining the regulated supply pressure is shown. This system consists of a positive displacement pump, a spring-loaded relief valve, and an accumulator. The pressure-flow characteristics of this system are shown in figure 1(b). The droop shown in the supply pressure (fig. 1(b)) is typical of that obtained from regulators such as the one shown in figure 1(a). The nozzle-system pressure shown is typical of a class of engine nozzle systems.

Zero-frequency output impedance. - The equation for output impedance for this system is (see appendix for derivation of this and subsequent equations)

$$\frac{\Delta P_o}{\Delta Q_o} = \frac{2(P_s - P_o)}{Q_o} \tag{2}$$

In view of the characteristic pressure variations shown in figure 1(b), equation (2) shows that the output impedance decreases as the system flow increases. Increased flow is obtained at increased throttle positions (increased throttle areas). Thus, output impedance is decreased as the throttle area is increased. This effect is shown in figure 1(c), where lines of constant throttle position are plotted on the pressure-flow plane. The slope of a constant-throttle-position curve at any point is the output impedance at that point. Equation (1)(appendix) indicates that a given value of output impedance can be obtained by maintaining the supply pressure sufficiently above the output pressure.

Dynamic characteristics. - If the compressibility and mass of the fluid in the system are not considered, the output flow is equal to the flow through the throttle at all throttle-variation frequencies. Therefore, if the supply pressure is maintained constant at all throttle-variation frequencies, the output impedance is independent of the throttle-variation frequency. However, in the practical case compressibility of the fluid in the conduit between the throttle and the system discharge orifice (engine nozzle system) causes a lag in the response of output flow to a change in throttle position. This effect manifests

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itself as a reduction of output impedance at high frequencies. The equation for the characteristic time constant that defines this effect is

$$\tau_c = \frac{\frac{(\Delta P_o)}{(\Delta Q_o)} Q_o V}{(P_s - P_C)B} \tag{75}$$

It is apparent from equation (75) that high-performance systems (characterized by high values of zero-frequency output impedance $\Delta P_o/\Delta Q_o$ and small values of time constant $\tau$) can be obtained only with small values of the conduit volume $V$ and high values of the supply pressure $P_s$.

## Bypass System

<u>Description of system</u>. - A general arrangement of a throttle with a bypass regulator, which controls the pressure difference across the throttle by diversion of the excess flow back to the pump inlet, is shown in figure 2(a). The action of the regulator is such that the upstream throttle pressure (pump discharge pressure) is equal to the sum of the output pressure and the spring bias. Consequently, the pressure drop across the throttle is the spring bias. These characteristics are shown graphically in figure 2(b), which illustrates the variation of pressures within the system with output flow.

<u>Zero-frequency output impedance</u>. - The equation for the output-flow impedance of this system is

$$\frac{\Delta P_o}{\Delta Q_o} = \frac{4P_t A_p wD(P_o + P_t - P_d)^{3/2}}{KQ_o(Q_G - Q_o)} \tag{19}$$

For a given regulator with fixed design parameters, the impedance varies with output pressure $P_o$ and flow $Q_o$. The denominator has a maximum value of $Q_o = Q_G/2$ and is equal to zero at $Q_o = 0$ and $Q_o = Q_G$. The numerator is dependent on $P_o$, which increases with $Q_o^2$. Thus the numerator increases approximately as $Q_o^3$. These effects are shown graphically in figure 2(c), where lines of constant throttle position are plotted on the pressure-flow plane. The term $\frac{P_t A_p w}{K}$ is the design parameter of the regulator. It is possible to design the regulator so that this term becomes large enough to make the variations of impedance with $P_o$ and $Q_o$ of no practical significance. The limitation on the maximum value of this term is discussed in the following section.

Dynamic characteristics. - If the bypass area could at every instant have a value such that the pressure drop across the throttle were constant regardless of the rapidity of throttle movement, the response of throttle flow to throttle position would be instantaneous. However, the area of the bypass valve is adjusted by a piston which, in moving in response to pressure changes, pumps a flow from one side of the throttle to the other. This effect causes a lag in the response of output flow to a change in throttle position and a lag in correcting the output flow for a disturbance in output pressure. The latter effect manifests itself as a reduction of output impedance at high frequencies. If the mass of the piston and valve is small, the response of valve movement to pressure can be described by a first-order system. Such a system is completely described by a time constant. When the piston and valve have appreciable mass, the response is approximately second order. The significance of the piston and valve mass on the response of the system can be evaluated by examination of the damping ratio of the system. 3613

The equations for time constant and damping ratio, respectively, of this system are

$$\tau = \frac{A_p}{Dw(P_o + P_t - P_d)^{3/2}} \left[P_o + P_t - P_d + (P_o - P_C)\left(\frac{Q_G}{Q_o} - 1\right)\right] \tag{62}$$

and

$$\delta = \frac{1 + \left(\frac{P_o - P_C}{P_o + P_t - P_d}\right)\left(\frac{Q_G}{Q_o} - 1\right)}{\sqrt{\frac{2MwD[Q_o(P_C - P_d) + Q_G(P_o + P_t - P_C)]}{A_p^3 P_t (P_o + P_t - P_d)^{1/2}}}} \tag{59}$$

An examination of equation (59) indicates that the relation of damping ratio to pressure and flow is quite complex. It does not appear practical to utilize the damping ratio in the usual design sense for second-order systems (i.e., $\delta = 0.7$), because the system is not regular enough to rely on resonance for improving the system response. In order to extend the usable amplitude response of a first-order system to a frequency corresponding to the usable amplitude response of a 0.7 damped second-order system, it is necessary to design a system on the basis of a smaller value of time constant (eq. (62)) and a sufficiently high value of damping ratio to ensure a substantially first-order response. Examination of equations (62) and (59) shows that this condition can only be achieved through the use of very light-weight valve elements.

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Equation (62) shows that small values of time constant are obtained through the combination of small values of piston area $A_p$, large valve widths $w$, large throttle pressure drops $P_t$, and high output pressures $P_o$. It should also be noted that the independent variable, pump flow $Q_G$, affects the time constant. In order to keep the time constant small, the pump should not be any larger than necessary to supply the maximum desired output flow.

The preceding analysis does not take into account the effect of fluid compressibility in the conduit between the throttle and the system discharge orifice. However, in most installations the connecting-line geometry is of such complex nature that, when a compressibility effect of significant magnitude exists, there is little interaction with the motion of the bypass valve. The compressibility effect then appears as a lag in series with the lag introduced by the bypass valve. If the compressibility effect is of significant magnitude, the effect will be apparent at frequencies below the break frequency determined by equation (62). Below this frequency the output impedance of the bypass system is essentially infinite, and the time constant of the conduit is determined by the fluid compressibility and the flow resistance of the discharge orifice of the system. The equation for this time constant is

$$\tau_c = \frac{2V(P_o - P_C)}{BQ_o} \tag{77}$$

Fluid compressibility in the conduit between the pump and the bypass line reduces the damping effect introduced by the constant-flow pump. The pump should therefore be located as close as possible to the bypass valve, and the conduit should be short and rigid.

## Reducing-Valve System

Description of system. - A general arrangement of a throttle with a reducing-valve regulator that controls the pressure difference across the throttle by reducing the pressure to the throttle is shown in figure 3(a). The action of the regulator is such that the upstream throttle pressure is equal to the sum of the output pressure and the spring bias. Consequently, the pressure drop across the throttle is the spring bias. The difference between upstream throttle pressure and supply pressure is the drop across the reducing valve. These characteristics are shown graphically in figure 3(b), which illustrates the variation of pressures within the system with output flow.

Zero-frequency output impedance. - The equation for the output-flow impedance of this system is

$$\frac{\Delta P_o}{\Delta Q_o} = \frac{4P_t A_p w D(P_s - P_t - P_o)^{3/2}}{KQ_o^2} \tag{12}$$

For a given regulator with fixed design parameters, the output impedance varies with output pressure $P_o$ and flow $Q_o$. It may be seen from equation (12) that the impedance reduces rapidly as the output pressure $P_o$ approaches $(P_s - P_t)$. It should be noted, however, that the impedance does not become zero but becomes equal to the equivalent throttle impedance curve at the point at which the reducing valve reaches its limiting area. These effects are shown graphically in figure 3(c), where lines of constant throttle position are plotted on the pressure-flow plane. The impedance may be maintained at a sufficiently high value by setting the supply pressure $P_s$ at a high value relative to the output pressure $P_o$. The term $\frac{P_t A_p w}{K}$ is the design parameter of the regulator. Although the impedance may be raised by proper choice of values for this term, the range of values is limited by dynamic considerations as shown in the following section. 3613

Dynamic characteristics. - In the case of the reducing-valve system, the pumping-action and mass effects of the piston and valve are similar to the effects described in relation to the bypass system. The equations for the time constant and damping ratio, respectively, of this system are

$$\tau = \frac{A_p(P_s - P_t - P_C)}{Dw(P_s - P_t - P_o)^{3/2}} \tag{43}$$

and

$$\delta = \frac{1 + \dfrac{P_o - P_C}{P_s - P_t - P_o}}{\sqrt{\dfrac{2MwDQ_o(P_s - P_C)}{A_p^3 P_t (P_s - P_t - P_o)^{1/2}}}} \tag{40}$$

The discussion concerning the relation of the damping ratio to the response of the bypass system also applies to this system. Thus, high-performance systems require the use of light-weight piston and valve parts. The effect of piston area $A_p$ and valve width $w$ on the time constant is the same as that for the bypass type; however, in the case of the reducing-valve system the throttle pressure drop and output pressure have the opposite effect. Equation (43) indicates that the time constant is reduced by setting the supply pressure $P_s$ as high as possible.

The effect of fluid compressibility in the conduit between the throttle and the system discharge orifice is the same as the effect described in the case of the bypass system. However, in the case of the reducing-valve system the requirement of a relatively close pump location is eliminated because of the use of a regulated supply pressure which entails the use of an accumulator. This accumulator should be located as near the reducing valve as possible and should have a short connecting line with as large a diameter as practicable to minimize the inductive effect between the accumulator and valve.



## PHYSICAL CONSTANTS OF SYSTEMS

Data were obtained on systems conforming to the schematics of figures 2 and 3. The physical constants of these systems were as follows:

| | |
|---|---|
| $A_p$, sq in. | 0.686 |
| $w$, in. | 1.0 |
| $K$, lb/in. | 68 |
| $M$ (bypass type), (lb)(sec$^2$)/in. | $196\times10^{-6}$ |
| $M$ (reducing type), (lb)(sec$^2$)/in. | $210\times10^{-6}$ |
| $P_c$, lb/sq in. | 0 |
| $P_d$, lb/sq in. | 60 |
| $P_o$, lb/sq in. | 0 to 300 |
| $P_s$, lb/sq in. | 600 |
| $P_t$, lb/sq in. | 100 |
| $Q_G$, cu in./sec | 80 |
| $Q_o$, cu in./sec | 0 to 60 |



The value of D for the fluid used was approximately 100 sq in./sec-$\sqrt{\text{lb}}$. The viscosity of the fluid was 1 centipoise.

## PROCEDURE AND INSTRUMENTATION

The two functions of interest in the determination of the dynamics of these units were the transfer functions of throttle position to input signal and fuel flow to throttle position. The throttle was positioned by an electrohydraulic servomotor responding to an input voltage. The servomotor system consisted of a commercial electrically operated servovalve, a piston integral with the throttle shaft, and a differential-transformer throttle-position pickup.

The frequency response of the throttle servomotor system is shown in figure 4. Up to an amplitude of 0.025 inch (or 10 percent of max. stroke), the response can be maintained level to 100 cycles per second by adjustment of the input-voltage amplitude. At maximum amplitude (0.25 in. or 100 percent of max. stroke), the response can be maintained level to 35 cycles per second.

The throttle position was measured by the differential-transformer displacement pickup incorporated in the servomotor system which used a 5000-cycle-per-second carrier. With this carrier frequency, the frequency response of the displacement pickup is flat to over 500 cycles per second.

Changes in flow were indicated by measuring the pressure difference across an orifice at the outlet of the control. Because the orifice discharged into atmospheric pressure, only the pressure upstream of the orifice $P_o$ was measured. Thus, the response of output flow $Q_o$ to throttle position becomes a function of the response of output pressure $P_o$ to throttle position. 3613

The pressure pickup used was a flush-diaphragm type, which was mounted with the diaphragm in the flowing stream to avoid any line effects. The response of this unit extends to 1000 cycles per second. The voltages resulting from the position and pressure pickups were observed and recorded together with input voltage on a cathode-ray oscilloscope and an oscillograph. The frequency response of the oscillograph extends to 300 cycles per second.

The steady-state measurements of output flow as a function of output pressure at constant throttle positions were made by varying the output restriction. The resulting pressure and flow variations were measured with pressure gages and rotameters. Multiple units were used so that the maximum instrument error was ±3 percent. A photograph of the fuel-control unit and associated test components is shown in figure 5.

Frequency-response runs from 1 to 100 cycles per second were made at a number of steady-state fuel-flow levels and flow amplitudes. Transient responses were also observed at a number of steady-state fuel flows and step amplitudes.

## RESULTS AND DISCUSSION

### Throttle System

The transient response, the frequency response, and the pressure-flow characteristics of the throttle system are shown in figure 6. A typical response of pressure (flow) to throttle position is shown in figure 6(a). The recording does not show any perceptible lag between pressure and throttle position. The frequency response of the throttle system is shown in figure 6(b). These data show an attenuation at high frequencies substantially as predicted by the analysis of the compressibility effect. The average break frequency is about 80 cycles per second. The theoretical break frequency, calculated by means of equation (75), corresponds to the 80-cycle-per-second value for a bulk modulus of 100,000 pounds per square inch. This value of bulk modulus is about half the value usually found in liquids of the type used; however, the lower value is possible in view of the probability that entrained air and vapor are released by the high throttle pressure drop. The data also indicate a resonance in the system at a frequency beyond 300 cycles per second. This resonance appears to be caused by the inductive effect of the fluid at the throat of the discharge orifice in combination with compressibility of the fluid between the throttle and the discharge orifice.

The slope of a constant-throttle-position line in figure 6(c) is the output impedance of the throttle system at that point and illustrates the reason why a throttle system is unsatisfactory in systems where the nozzle output pressure is subject to variations. At a flow of 6000 pounds per hour and a manifold pressure of 300 pounds per square inch, a change in nozzle output pressure of 1 pound per square inch will cause a flow change of about 10 pounds per hour. This system is also subject to flow changes with inlet pressure variations.

## Bypass System

The transient response, frequency response, and pressure-flow characteristics of the bypass system are shown in figure 7. The response of pressure (flow) to throttle position shown in figure 7(a) for a particular value of steady-state flow and transient amplitude is typical of the transient response for a wide range of steady-state flows and transient amplitudes. The frequency response of the bypass control shown in figure 7(b) for a number of steady-state flows and amplitudes indicates a range of break frequencies of about 25 to 50 cycles per second. This compares with a theoretical range of first-order break frequencies of 60 to 250 cycles per second.

The steady-state output impedance is indicated in figure 7(c). The values of impedance involved are too high to make any quantitative comparison with calculated values. A comparison of figures 7(c) and 6(c) shows that the bypass control system has an output impedance about 100 times as high as the throttle system at the higher flows. The impedance falls off in direct proportion to the reduction of amplitude ratio with frequency (fig. 7(b)).

## Reducing-Valve System

The transient response, frequency response, and pressure-flow characteristics of the reducing-valve system are shown in figure 8. The response of pressure (flow) to throttle position shown in figure 8(a) for a particular value of steady-state flow and transient amplitude is typical of the transient response for a wide range of steady-state flows and transient amplitudes. The frequency response shown in figure 8(b) for a number of steady-state flows and amplitudes indicates break frequencies of about 150 cycles per second and significant output amplitude at 300 cycles per second. This response is in close agreement with the values indicated by the linearized analysis. The phase shift shows more than a 45° lag at 150 cycles per second. This situation might be expected because the calculated values of damping ratio give values of approximately unity. The indication of steady-state output impedance is shown in figure 8(c). For all practical purposes the slope of these curves in the usable range is infinite.



## CONCLUDING REMARKS

The addition of a differential-pressure-regulating valve element to a throttle system results in a system in which the output flow is essentially independent of output pressure (high output impedance) and is a function only of throttle position. By the use of light-weight and properly sized valve elements, these characteristics can be maintained for output-pressure-variation or throttle-variation frequencies as high as 100 cycles per second.

The differential-pressure-regulating valve may be either the bypass or reducing-valve type. The selection of the type of regulation will depend on the system in which it is installed. The bypass system has the advantage of acting as the relief valve for the system. It has the disadvantage of requiring a relatively close location of the pump to the control unit and to the system in order to keep the stray capacitance in the system low and preserve the response. The reducing-valve system has the advantage of allowing a remote pump location but requires the use of additional components in the system (relief valve and accumulator).

The experimental results show that linearized analysis provides an adequate description of the system dynamic response. Of the three systems tested, the reducing-valve system showed closest agreement with linearized analysis and responded to the highest frequencies. The response of this system was adequate to 150 cycles per second and usable to 300 cycles per second.

Lewis Flight Propulsion Laboratory
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## APPENDIX - DERIVATION OF EQUATIONS

Zero-frequency output impedance of throttle system (fig. 1). - In steady state the output flow is equal to the flow through the throttle. The output flow can then be expressed in terms of the throttle area and the pressure drop across the throttle as

$$Q_o = DA_t(P_s - P_o) \tag{1}$$

3613 Differentiating equation (1) with respect to $Q_o$ and making suitable substitutions yield the output impedance at zero frequency:

$$\frac{\Delta P_o}{\Delta Q_o} = \frac{2(P_s - P_o)}{Q_o} \tag{2}$$

Zero-frequency output impedance of reducing-valve system (fig. 3). - For the case of regulation with small load error, the steady-state deviation of the regulated pressure difference across the throttle that corresponds to a change in valve area is

$$\Delta P_t = -\frac{K\Delta A_v}{A_p w} \tag{3}$$

The corresponding deviation in output flow is

$$\Delta Q_o = \frac{dQ_o}{dP_t}\Delta P_t \tag{4}$$

The steady-state output flow may be expressed as

$$Q_o = DA_t\sqrt{P_t} \tag{5}$$

If changes in $Q_o$ that are small compared with the steady-state value are considered, equation (5) yields

$$\frac{dQ_o}{dP_t} = \frac{Q_o}{2P_t} \tag{6}$$

The steady-state output flow may be expressed by a second equation

$$Q_o = DA_v\sqrt{P_s - P_r} \tag{7}$$

Equation (7) yields

$$\frac{dP_r}{dA_v} = \frac{2D(P_s - P_r)^{3/2}}{Q_o} \tag{8}$$

From equation (8),

$$\Delta A_v = \frac{Q_o \Delta P_r}{2D(P_s - P_r)^{3/2}} \tag{9}$$

The change in $P_r$ and $P_o$ will be large compared with the deviation of $P_t$; hence, the relation

$$\Delta P_r = \Delta P_o + \Delta P_t$$

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yields

$$\Delta P_r \cong \Delta P_o \tag{10}$$

Combining equations (3), (4), (6), (9), and (10) gives

$$\frac{\Delta P_o}{\Delta Q_o} = - \frac{4P_t A_p wD(P_s - P_r)^{3/2}}{KQ_o^2} \tag{11}$$

or

$$\frac{\Delta P_o}{\Delta Q_o} = - \frac{4P_t A_p wD(P_s - P_t - P_o)^{3/2}}{KQ_o^2} \tag{12}$$

Zero-frequency output impedance of bypass system (fig. 2). - For the case of regulation with small load error, the steady-state deviation of the pressure difference across the throttle corresponding to a change in control-valve area is

$$\Delta P_t = \frac{K \Delta A_v}{A_p w} \tag{13}$$

The steady-state flow through the control valve is

$$Q_v = DA_v \sqrt{P_r - P_d} \tag{14}$$

If changes in $Q_v$ that are small compared with the steady-state value are considered, equation (14) yields

$$\frac{dP_r}{dA_v} = - \frac{2D(P_r - P_d)^{3/2}}{Q_v} \tag{15}$$

From equation (15),

$$\Delta A_v = -\frac{Q_v}{2D(P_r - P_d)^{3/2}} \Delta P_r \tag{16}$$

The change in $P_r$ and $P_o$ will be large compared with the deviation of $P_t$; hence

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$$\Delta P_r \cong \Delta P_o \tag{17}$$

Equations (4) and (6) apply without change; therefore, combining equations (4), (6), (13), (15), and (17) yields

$$\frac{\Delta P_o}{\Delta Q_o} = -\frac{4P_t A_p w D(P_r - P_d)^{3/2}}{K Q_v Q_o} \tag{18}$$

or

$$\frac{\Delta P_o}{\Delta Q_o} = -\frac{4P_t A_p w D(P_o + P_t - P_d)^{3/2}}{K Q_o (Q_G - Q_o)} \tag{19}$$

Dynamic characteristics of reducing-valve system (fig. 3). - If small disturbances are considered, the instantaneous deviation of regulated pressure from steady state may be written in terms of three principal equations. One equation obtained from the steady-state flow relations is

$$\Delta P_r = \left(\frac{dP_r}{dQ_t}\right) \Delta Q_t + \left(\frac{dP_o}{dQ_o}\right) \Delta Q_o \tag{20}$$

A second equation which expresses the effect of the control valve on the regulated pressure is

$$\Delta P_r = \left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v} \Delta Q_v + \left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v} \Delta A_v \tag{21}$$

The third equation, expressing the force equilibrium (neglecting drag and change in spring load), is

$$\Delta P_r = \left(\frac{dP_o}{dQ_o}\right) \Delta Q_o + \frac{M}{A_p^2} \Delta \dot{Q}_p \tag{22}$$

The equilibrium of flow into and out of chamber A (fig. 3) is

$$\Delta Q_t + \Delta Q_p - \Delta Q_v = 0 \tag{23}$$

The equilibrium of flow into and out of chamber B (fig. 3) is

$$\Delta Q_o - \Delta Q_p - \Delta Q_t = 0 \tag{24}$$

Noting that

$$\Delta Q_p = A_p \Delta \dot{x} \tag{25}$$

and

$$\Delta A_v = - w \Delta x \tag{26}$$

then

$$\Delta A_v = - \frac{w}{A_p} \int_0^t \Delta Q_p dt \tag{27}$$

Combining equations (20) and (24) yields

$$\Delta P_r = \left[\left(\frac{dP_r}{dQ_t}\right) + \left(\frac{dP_o}{dQ_o}\right)\right] \Delta Q_o - \left(\frac{dP_r}{dQ_t}\right) \Delta Q_p = 0 \tag{28}$$

Eliminating $\Delta P_r$ between equations (22) and (28) gives

$$\Delta Q_o = \frac{M}{A_p^2 \left(\frac{dP_r}{dQ_t}\right)} \Delta \dot{Q}_p + \Delta Q_p \tag{29}$$

Substituting equation (29) into equation (22) gives

$$\Delta P_r = \frac{M}{A_p^2} \left[1 + \frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{dP_r}{dQ_t}\right)}\right] \Delta \dot{Q}_p + \left(\frac{dP_o}{dQ_o}\right) \Delta Q_p \tag{30}$$

Combining equations (21), (27), and (30) yields

$$\Delta Q_v = \frac{M}{A_p^2\left(\frac{\partial P_r}{\partial A_v}\right)_{A_v}}\left[1 + \frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{dP_r}{dQ_t}\right)}\right]\Delta\dot{Q}_p + \frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\Delta Q_p + \frac{\left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v}}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\frac{w}{A_p}\int_0^t \Delta Q_p dt \tag{31}$$



From equations (23) and (24),

$$\Delta Q_o - \Delta Q_v = 0 \tag{32}$$

Substituting equations (29) and (31) into equation (32) yields

$$\frac{M}{A_p^2}\left\{-\frac{1}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\left[1 + \frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{dP_r}{dQ_t}\right)}\right] + \frac{1}{\left(\frac{dP_r}{dQ_t}\right)}\right\}\Delta\dot{Q}_p + \left[1 - \frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\right]\Delta Q_p - \left[\frac{\left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v}}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\frac{w}{A_p}\right]\int_0^t \Delta Q_p dt = 0 \tag{33}$$



From equation (33), the system damping ratio is

$$\delta = \frac{\left[1 - \frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\right]}{2\sqrt{\frac{Mw}{A_p^3}\frac{\left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v}}{-\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\left\{\frac{1}{\left(\frac{dP_r}{dQ_t}\right)} - \frac{1}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\left[1 + \frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{dP_r}{dQ_t}\right)}\right]\right\}}} \tag{34}$$

Evaluating the derivatives gives the following relations:

$$\left(\frac{dP_r}{dQ_t}\right) = \frac{2P_t}{Q_o} \tag{35}$$

$$\left(\frac{dP_o}{dQ_o}\right) = \frac{2(P_o - P_C)}{Q_o} \tag{36}$$

$$\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v} = -\frac{2(P_s - P_r)}{Q_o} \tag{37}$$

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$$\left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v} = \frac{2D(P_s - P_r)^{3/2}}{Q_o} \tag{38}$$

Substituting the derivatives into equation (34) yields

$$\delta = \frac{1 + \dfrac{P_o - P_C}{P_s - P_r}}{\sqrt{\dfrac{2MwDQ_o(P_s - P_C)}{A_p^3 P_t (P_s - P_r)^{1/2}}}} \tag{39}$$

Noting that $P_r \approx P_o + P_t$, equation (39) becomes

$$\delta \approx \frac{1 + \dfrac{P_o - P_C}{P_s - P_t - P_o}}{\sqrt{\dfrac{2MwDQ_o(P_s - P_C)}{A_p^3 P_t (P_s - P_t - P_o)^{1/2}}}} \tag{40}$$

In the case of the overdamped regulator in which

$$\frac{M}{A_p^2} \cong 0$$

equation (33) reduces to

$$\left[1 - \frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\right]\Delta\dot{Q}_p - \left[\frac{\left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v}}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\frac{w}{A_p}\right]\Delta Q_p = 0 \tag{41}$$



From equation (41) the characteristic time constant of the system is

$$\tau = \left[\frac{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v} - \left(\frac{dP_o}{dQ_o}\right)}{-\left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v}}\right]\frac{A_p}{w} \tag{42}$$



Substituting the derivative into equation (42) gives

$$\tau = \frac{A_p(P_s - P_t - P_C)}{Dw(P_s - P_t - P_o)^{3/2}} \tag{43}$$

In the massless case there is no transient variation of $Q_t$; therefore equation (24) reduces to

$$\Delta Q_o = \Delta Q_p \tag{44}$$

Hence in the case of the massless piston, the output flow lags the throttle position with a response characterized by the time constant $\tau$ (eq. (43)).

Dynamic characteristics of bypass system (fig. 2). - Considering an ideal (leakless) pump, the equations defining the instantaneous deviation of the regulated pressure from steady state are identical with those previously written for the reducing-valve system (eqs. (20), (21), and (22)).

The equilibrium of flow into and out of chamber A (fig. 2) is

$$\Delta Q_t + \Delta Q_p + \Delta Q_v = 0 \tag{45}$$

The equilibrium of flow into and out of chamber B (fig. 2) is

$$\Delta Q_o - \Delta Q_p - \Delta Q_t = 0 \tag{46}$$

In the case of the bypass system,

$$\Delta A_v = wx \tag{47}$$

Hence,

$$\Delta A_v = \frac{w}{A_p} \int_0^t \Delta Q_p dt \tag{48}$$

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Combining equations (20) and (46) yields

$$\Delta P_r = \left[\left(\frac{dP_r}{dQ_t}\right) + \left(\frac{dP_o}{dQ_o}\right)\right] \Delta Q_o - \left(\frac{dP_r}{dQ_t}\right) \Delta Q_p \tag{49}$$

Eliminating $\Delta P_r$ between equations (22) and (49) gives

$$\Delta Q_o = \frac{M}{A_p^2 \left(\frac{dP_r}{dQ_t}\right)} \Delta \dot{Q}_p + \Delta Q_p \tag{50}$$

Substituting equation (50) into equation (49) gives

$$\Delta P_r = \frac{M}{A_p^2} \left[1 + \frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{dP_r}{dQ_t}\right)}\right] \Delta \dot{Q}_p + \left(\frac{dP_o}{dQ_o}\right) \Delta Q_p \tag{51}$$

Combining equations (21), (48), and (51) yields

$$\Delta Q_v = \frac{M}{A_p^2 \left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}} \left[1 + \frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{dP_r}{dQ_t}\right)}\right] \Delta \dot{Q}_p + \frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{dP_r}{dQ_v}\right)} \Delta Q_p - \frac{\left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v}}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}} \frac{w}{A_p} \int_0^t \Delta Q_p dt \tag{52}$$

From equations (45) and (46)

$$\Delta Q_o + \Delta Q_v = 0 \tag{53}$$

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Substituting equations (50) and (52) into equation (53) yields

$$\frac{M}{A_p^2}\left\{\frac{1}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\left[1+\frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{dP_r}{dQ_t}\right)}\right]+\frac{1}{\left(\frac{dP_r}{dQ_t}\right)}\right\}\Delta\dot{Q}_p+\left[1+\frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\right]\Delta Q_p -$$

$$\left[\frac{\left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v}}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\frac{w}{A_p}\right]\int_0^t \Delta Q_p dt = 0 \tag{54}$$

From equation (54), the system damping ratio is

$$\delta = \frac{\left[1+\frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\right]}{2\sqrt{\frac{-Mw}{A_p^3}\frac{\left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v}}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\left\{\frac{1}{\left(\frac{dP_r}{dQ_t}\right)}+\frac{1}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\left[1+\frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{dP_r}{dQ_t}\right)}\right]\right\}}} \tag{55}$$

Evaluating the derivatives and noting that $Q_v = Q_G - Q_o$ give

$$\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v} = \frac{2(P_r - P_d)}{Q_G - Q_o} \tag{56}$$

$$\left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v} = \frac{-2D(P_r - P_d)^{3/2}}{Q_G - Q_o} \tag{57}$$

Substituting equations (35), (36), (56), and (57) into equation (55) gives

$$\delta = \frac{\left[1 + \left(\frac{P_o - P_C}{P_r - P_d}\right)\left(\frac{Q_G}{Q_o} - 1\right)\right]}{\sqrt{\frac{2MwD(P_r - P_d)^{1/2}}{A_p^3}\left\{\frac{Q_o}{P_t} + \frac{(Q_G - Q_o)}{(P_r - P_d)}\left[1 + \frac{(P_o - P_C)}{P_t}\right]\right\}}} \tag{58}$$



or

$$\delta = \frac{1 + \left(\frac{P_o - P_C}{P_o + P_t - P_d}\right)\left(\frac{Q_G}{Q_o} - 1\right)}{\sqrt{\frac{2MwD\left[Q_o(P_C - P_d) + Q_G(P_o + P_t - P_C)\right]}{A_p^3 P_t(P_o + P_t - P_d)^{1/2}}}} \tag{59}$$

In the case of the overdamped system in which

$$\frac{M}{A_p^2} \cong 0$$

equation (54) reduces to

$$\left[1 + \frac{\left(\frac{dP_o}{dQ_o}\right)}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\right]\Delta\dot{Q}_p - \left[\frac{\left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v}}{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v}}\frac{w}{A_p}\right]\Delta Q_p = 0 \tag{60}$$

From equation (60) the characteristic time constant of the system is

$$\tau = \left[\frac{\left(\frac{\partial P_r}{\partial Q_v}\right)_{A_v} + \left(\frac{dP_o}{dQ_o}\right)}{-\left(\frac{\partial P_r}{\partial A_v}\right)_{Q_v}}\right]\frac{A_p}{w} \tag{61}$$

Substituting the derivatives into equation (61) gives

$$\tau = \frac{A_p}{Dw(P_o + P_t - P_d)^{3/2}} \left[ P_o + P_t - P_d + (P_o - P_C)\left(\frac{Q_G}{Q_o} - 1\right)\right] \tag{62}$$

In the massless case there is no transient variation in $Q_t$; therefore equation (46) reduces to



$$\Delta Q_o = \Delta Q_p \tag{63}$$

Hence in the massless case the output flow lags the throttle position with a response characterized by the time constant $\tau$ (eq. (62)).

Compressibility time constant of throttle system (fig. 1). - If small disturbances are considered, the instantaneous deviation of the output pressure from steady state in the conduit between the throttle and the discharge orifice is defined by the following independent equations:

$$\Delta P_o = - \frac{dP_o}{dQ_t} \Delta Q_t \tag{64}$$

$$\Delta P_o = \frac{dP_o}{dQ_o} \Delta Q_o \tag{65}$$

$$\Delta P_o = - \frac{B}{V} dV \tag{66}$$

The change in volume $dV$ is related to the compressibility flow by the following relation:

$$dV = \int_0^t \Delta Q_c dt \tag{67}$$

Hence equation (66) may be written as

$$\Delta P_o = - \frac{B}{V} \int_0^t \Delta Q_c dt \tag{68}$$

The equilibrium of flow into and out of the conduit is

$$\Delta Q_t - \Delta Q_c - \Delta Q_o = 0 \tag{69}$$

Combining equations (64), (65), (66), and (69) yields an equation of the following form:

$$\tau_c \Delta\dot{Q}_o + \Delta Q_o = 0 \tag{70}$$

where

$$\tau_c = \frac{V}{B}\left[\frac{\left(\frac{dP_o}{dQ_o}\right)\left(\frac{dP_o}{dQ_t}\right)}{\left(\frac{dP_o}{dQ_o}\right) + \left(\frac{dP_o}{dQ_t}\right)}\right] \tag{71}$$

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Evaluating the derivatives gives the following relations:

$$\frac{dP_o}{dQ_t} = \frac{2(P_s - P_o)}{Q_o} \tag{72}$$

$$\frac{dP_o}{dQ_o} = \frac{2(P_o - P_c)}{Q_o} \tag{73}$$

Substituting the derivatives into equation (71) gives

$$\tau_c = \frac{2V(P_s - P_o)(P_o - P_c)}{B(P_s - P_c)} \tag{74}$$

Substituting equation (2) into equation (74) yields the following:

$$\tau_c = \frac{\left(\frac{\Delta P_o}{\Delta Q_o}\right) Q_o V}{(P_s - P_c)B} \tag{75}$$

Compressibility time constant of bypass and reducing-valve systems (figs. 2 and 3). - Based on the assumption that fluid compressibility in the conduit between the throttle and the discharge orifice does not affect the motion of the bypass valve (or reducing valve), the equations of the dynamic equilibrium in the conduit are the same as those in the case of the throttle system. At frequencies below the break frequency, defined by the characteristic time constant of the bypass system (or reducing-valve system), the effective value of the derivative $dP_o/dQ_t$ is the value of the zero-frequency output impedance. In this case the value of $dP_o/dQ_t$ may be considered to be infinite, and equation (71) reduces to

$$\tau_c = \frac{V\left(\frac{dP_o}{dQ_o}\right)}{B} \tag{76}$$

Substituting equation (73) into equation (76) gives

$$\tau_c = \frac{2V(P_o - P_C)}{BQ_o} \tag{77}$$



REFERENCE

1. Gold, Harold, and Otto, Edward W.: An Analytical and Experimental Study of the Transient Response of a Pressure-Regulating Relief Valve in a Hydraulic Circuit. NACA TN 3102, 1954.

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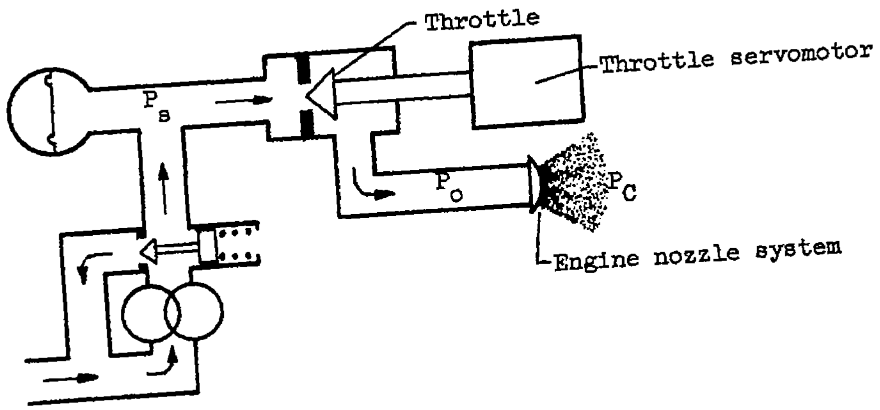
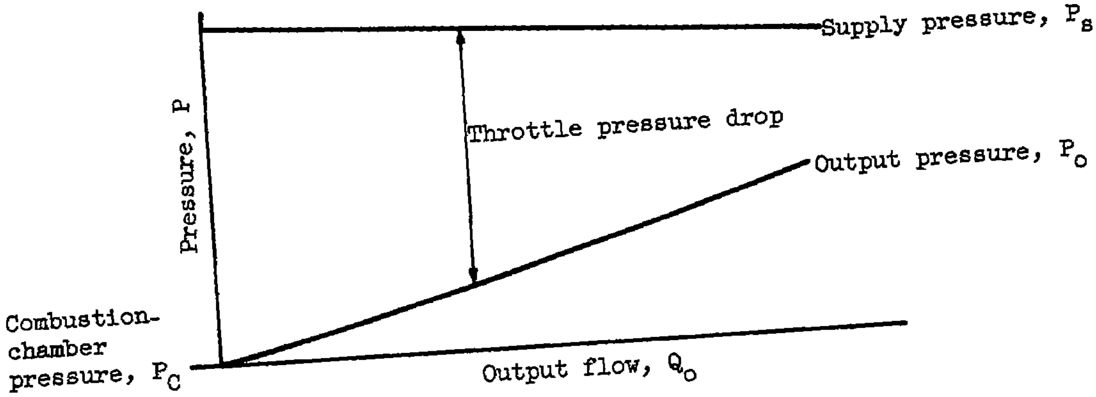
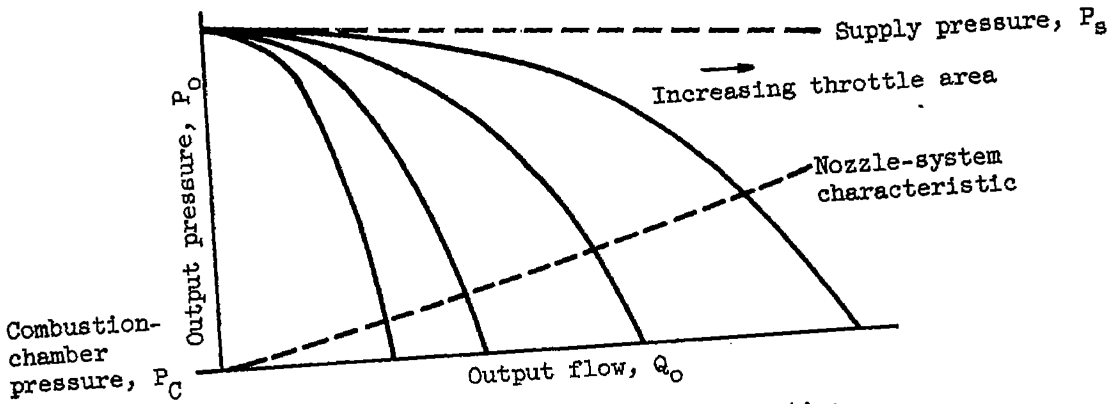



(a) Schematic diagram.




(b) Pressure-flow characteristics.




(c) Output-impedance characteristics.

Figure 1. - Schematic diagram and basic steady-state characteristics of throttle with regulated upstream pressure.

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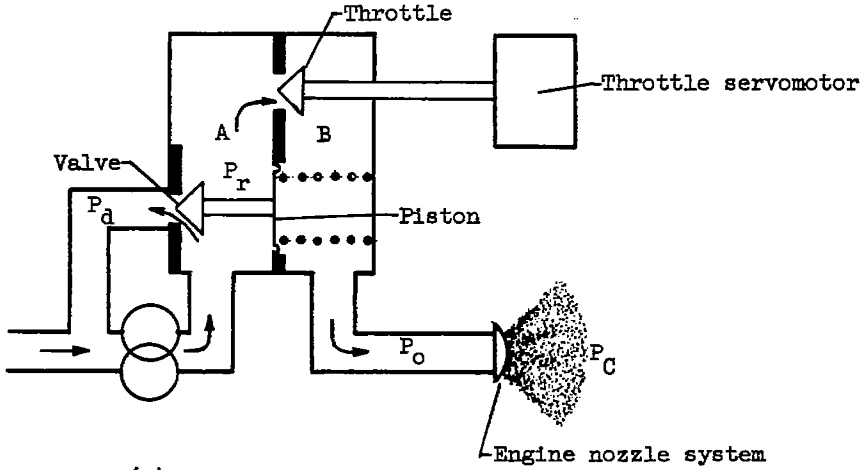
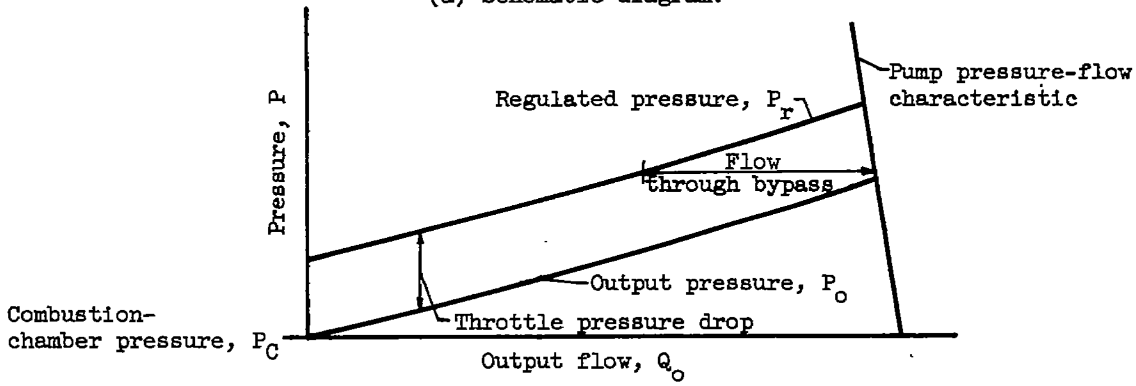
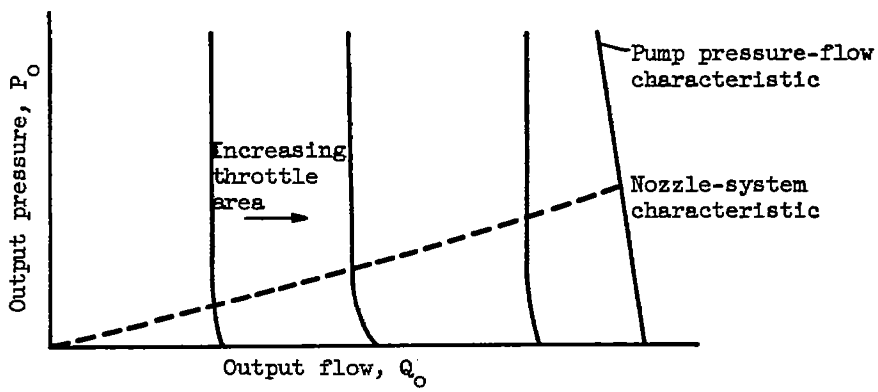



(a) Schematic diagram.

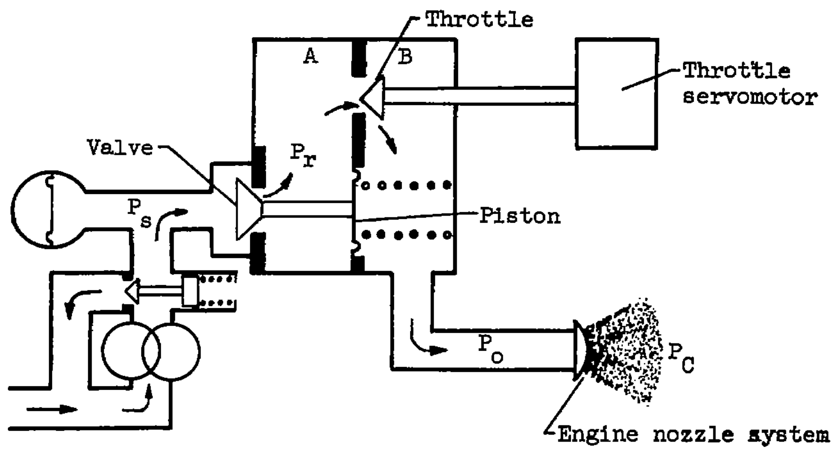



(b) Pressure-flow characteristics.

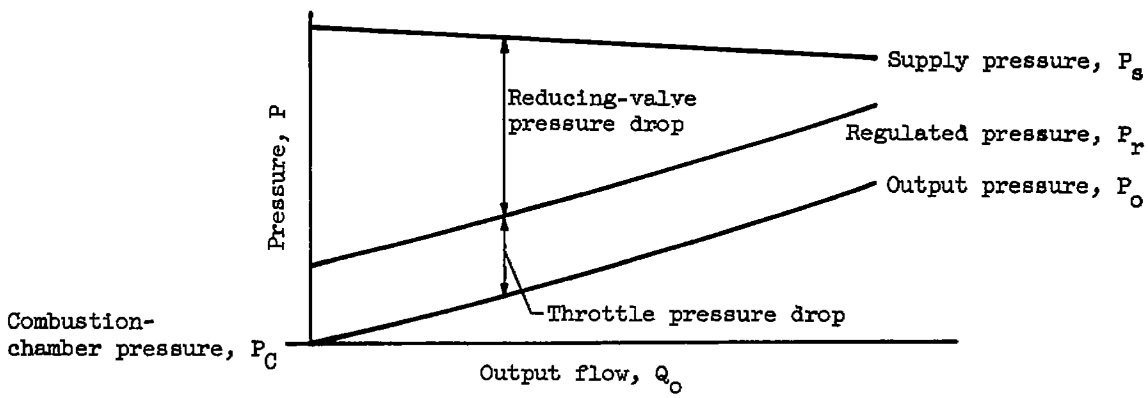



(c) Output-impedance characteristics.

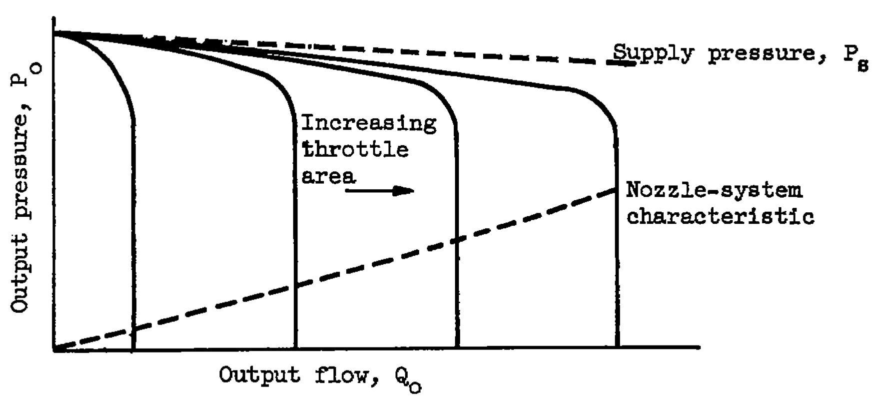
Figure 2. - Schematic diagram and basic characteristics of throttle plus bypass differential-pressure regulator.




(a) Schematic diagram.




(b) Pressure-flow characteristics.




(c) Output-impedance characteristics.

Figure 3. - Schematic diagram and basic characteristics of throttle plus reducing-valve differential-pressure regulator.

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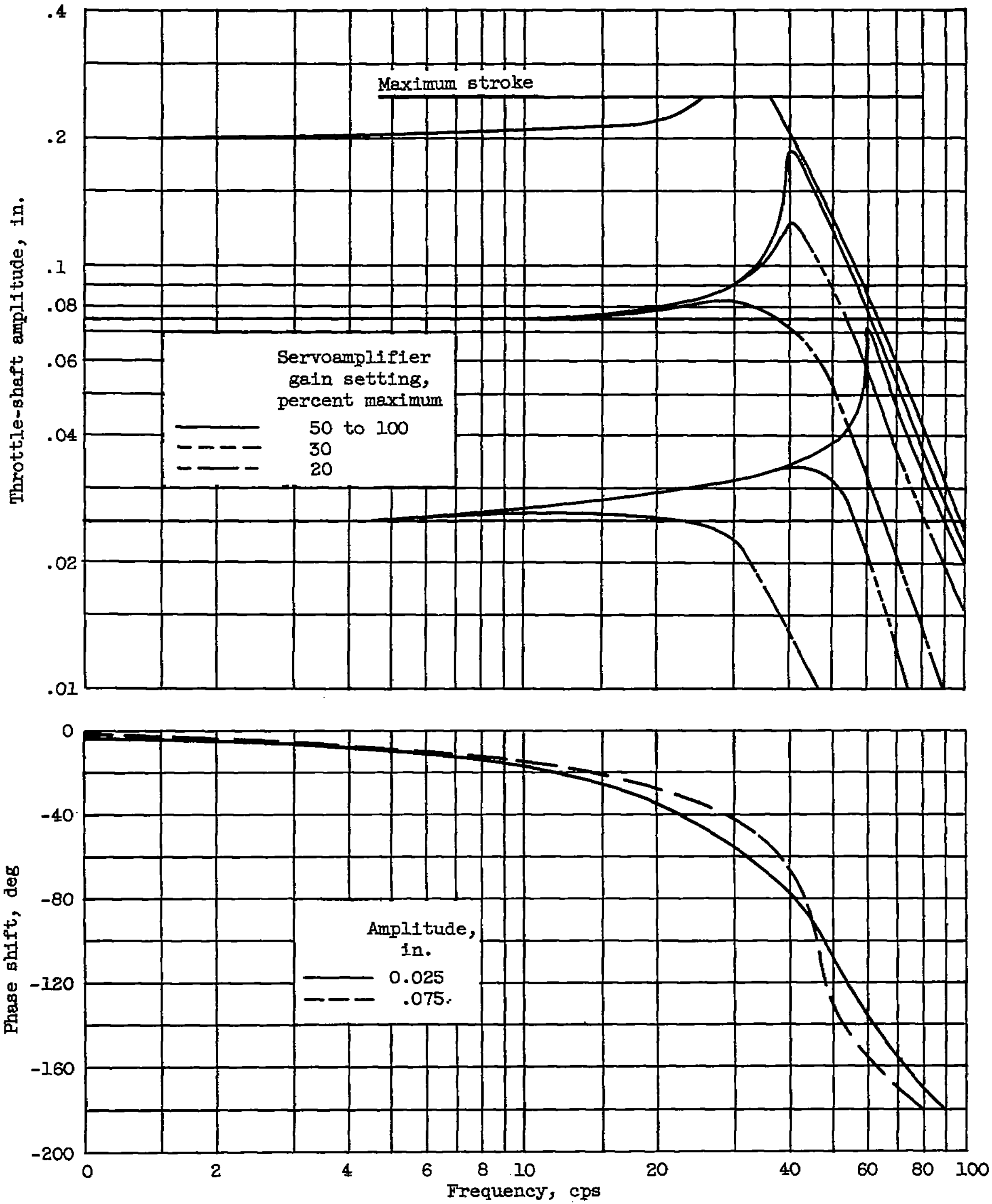


Figure 4. - Frequency-response characteristics of throttle-shaft servomotor.

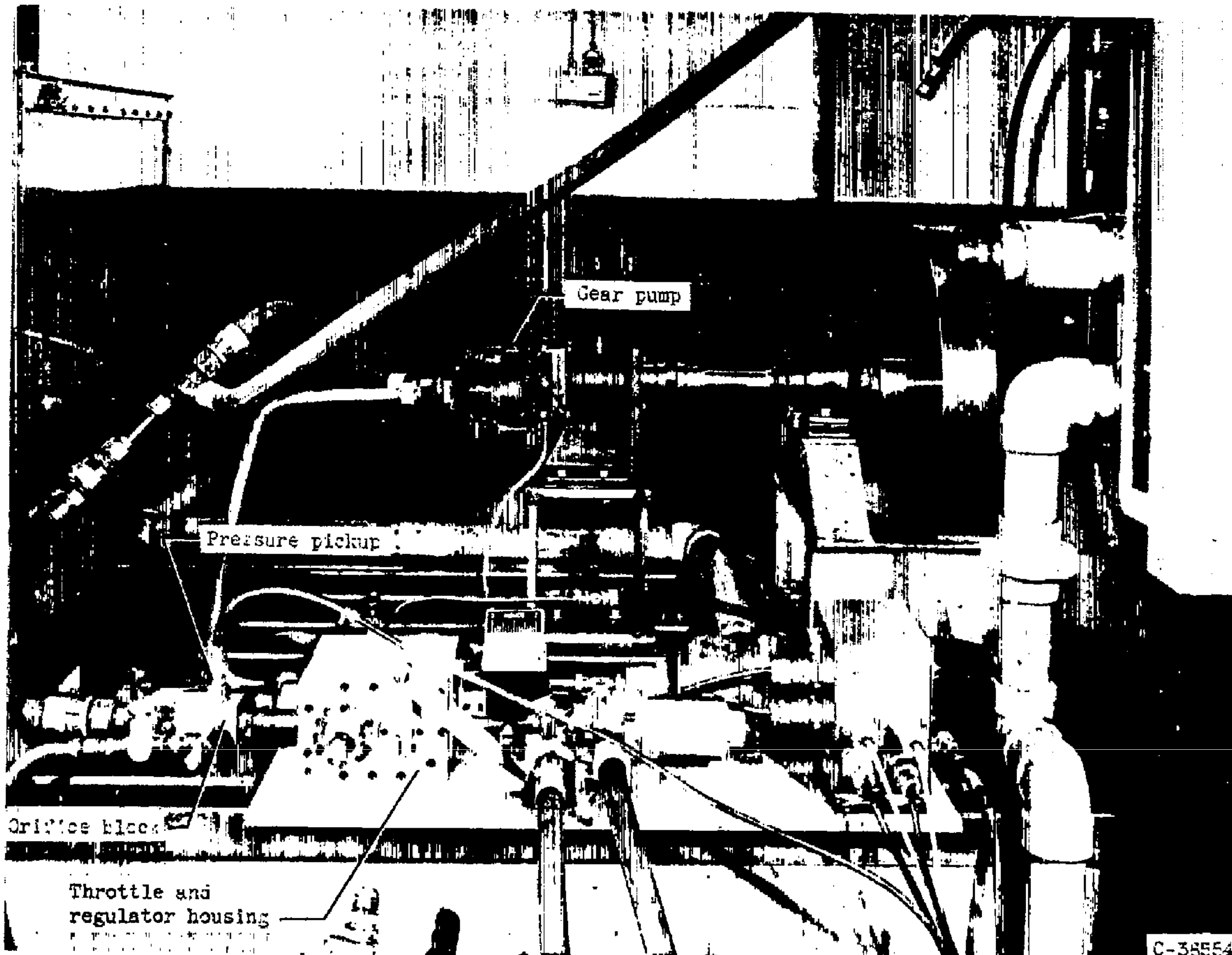


Figure 5. - Apparatus used in investigation of steady-state and dynamic characteristics of throttle-type fuel controls.

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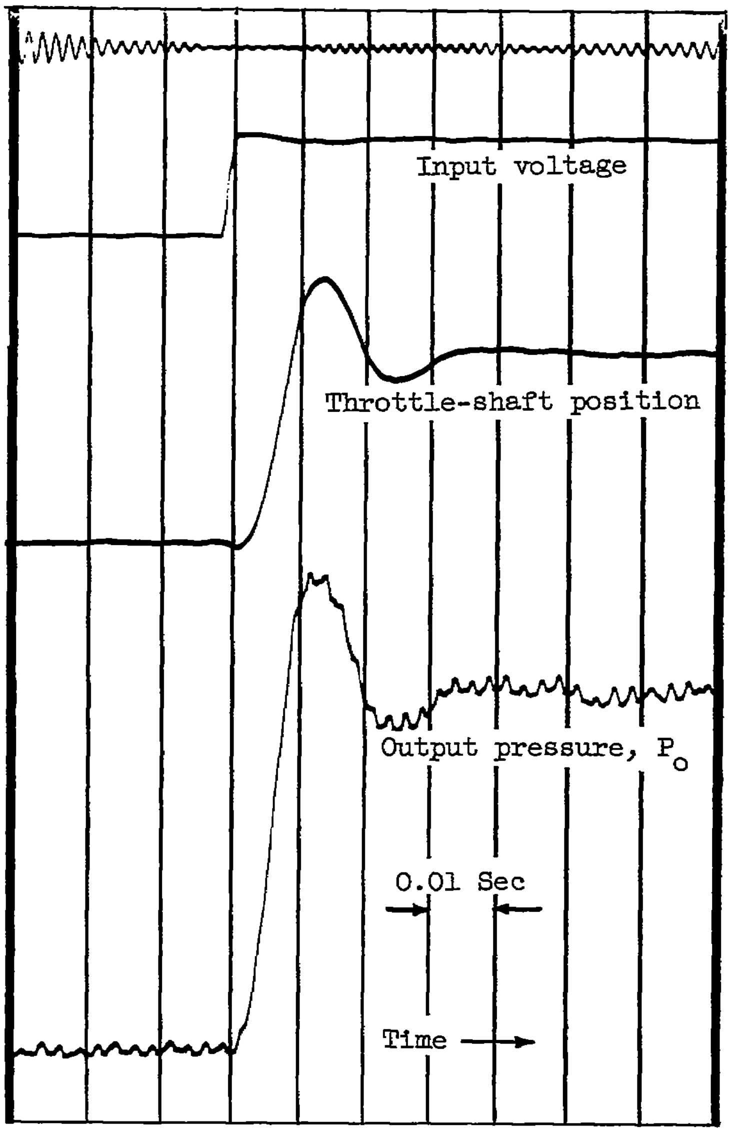


(a) Transient response. Initial fuel flow, 4000 pounds per hour; final fuel flow, 5000 pounds per hour.

Figure 6. - Throttle system.

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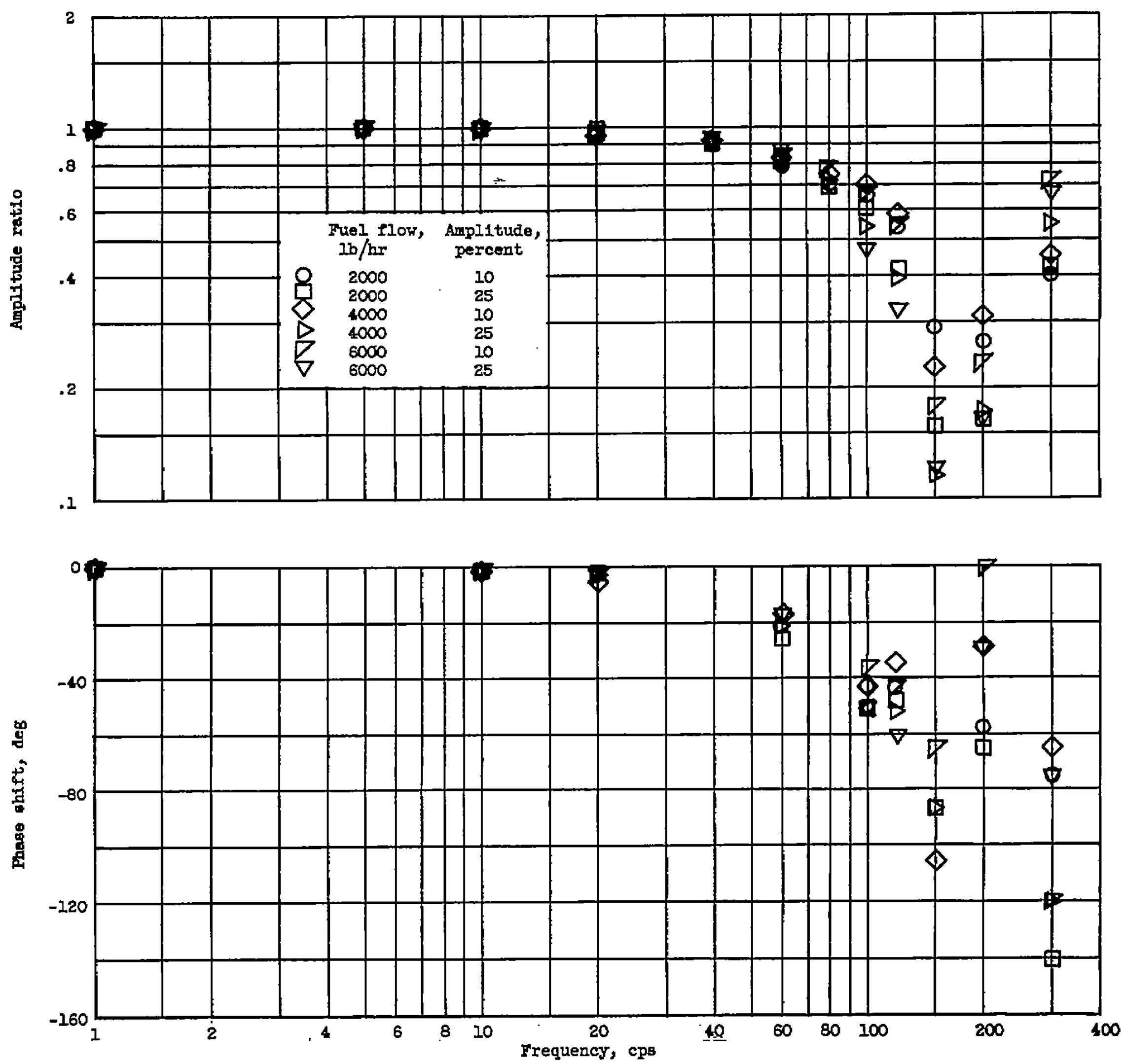
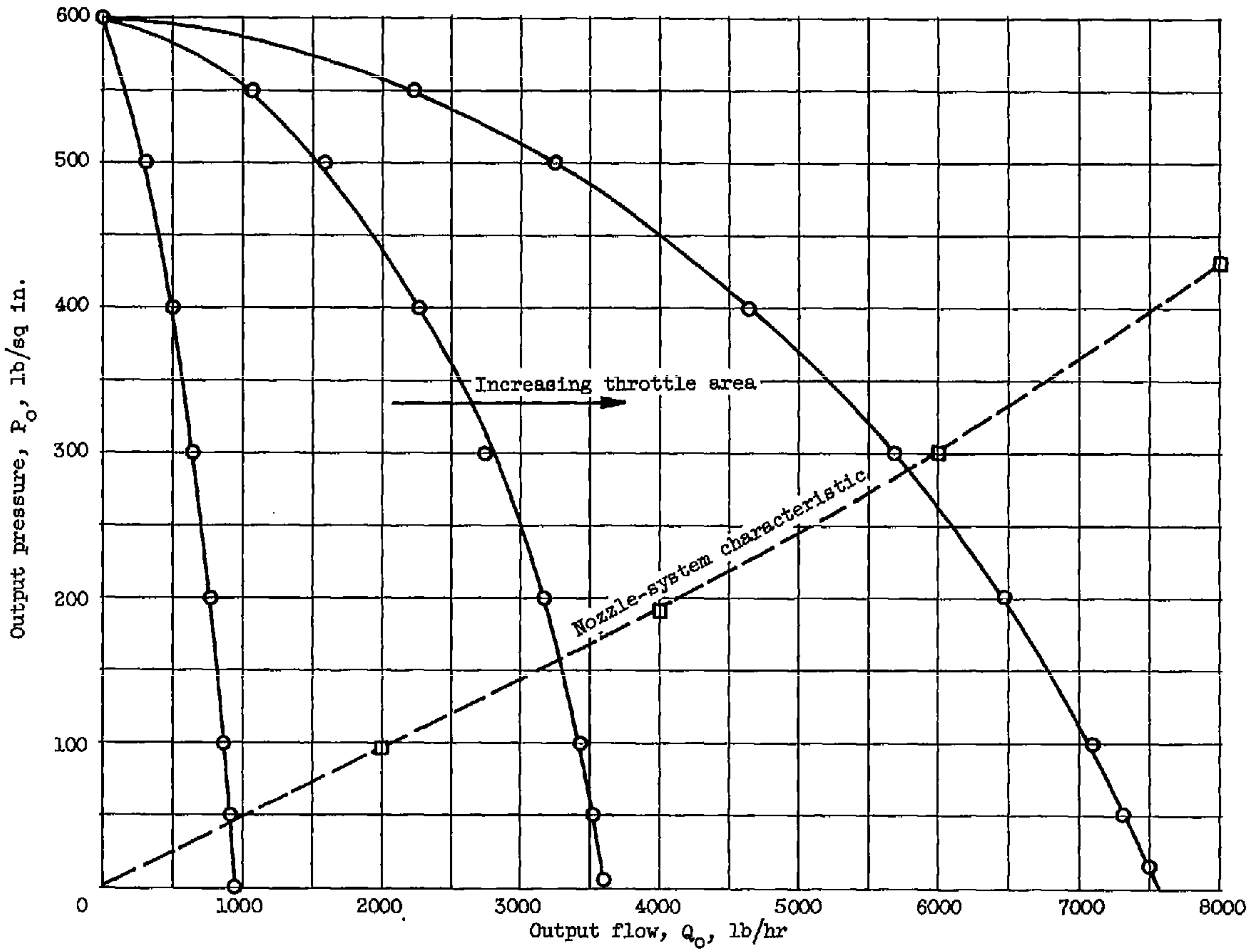



(b) Frequency response.

Figure 6. - Continued. Throttle system.




(c) Output-impedance characteristics.

Figure 6. - Concluded. Throttle system.

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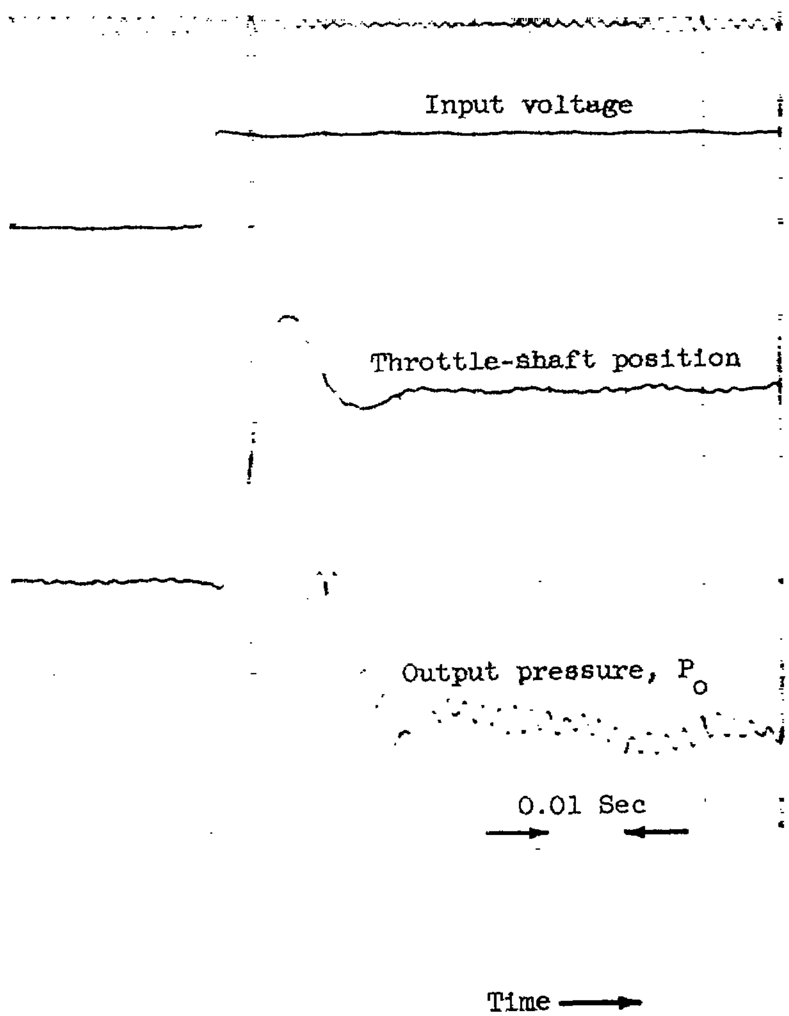


(a) Transient response. Initial fuel flow, 6000 pounds per hour; final fuel flow, 7500 pounds per hour.

Figure 7. - Throttle plus bypass system.

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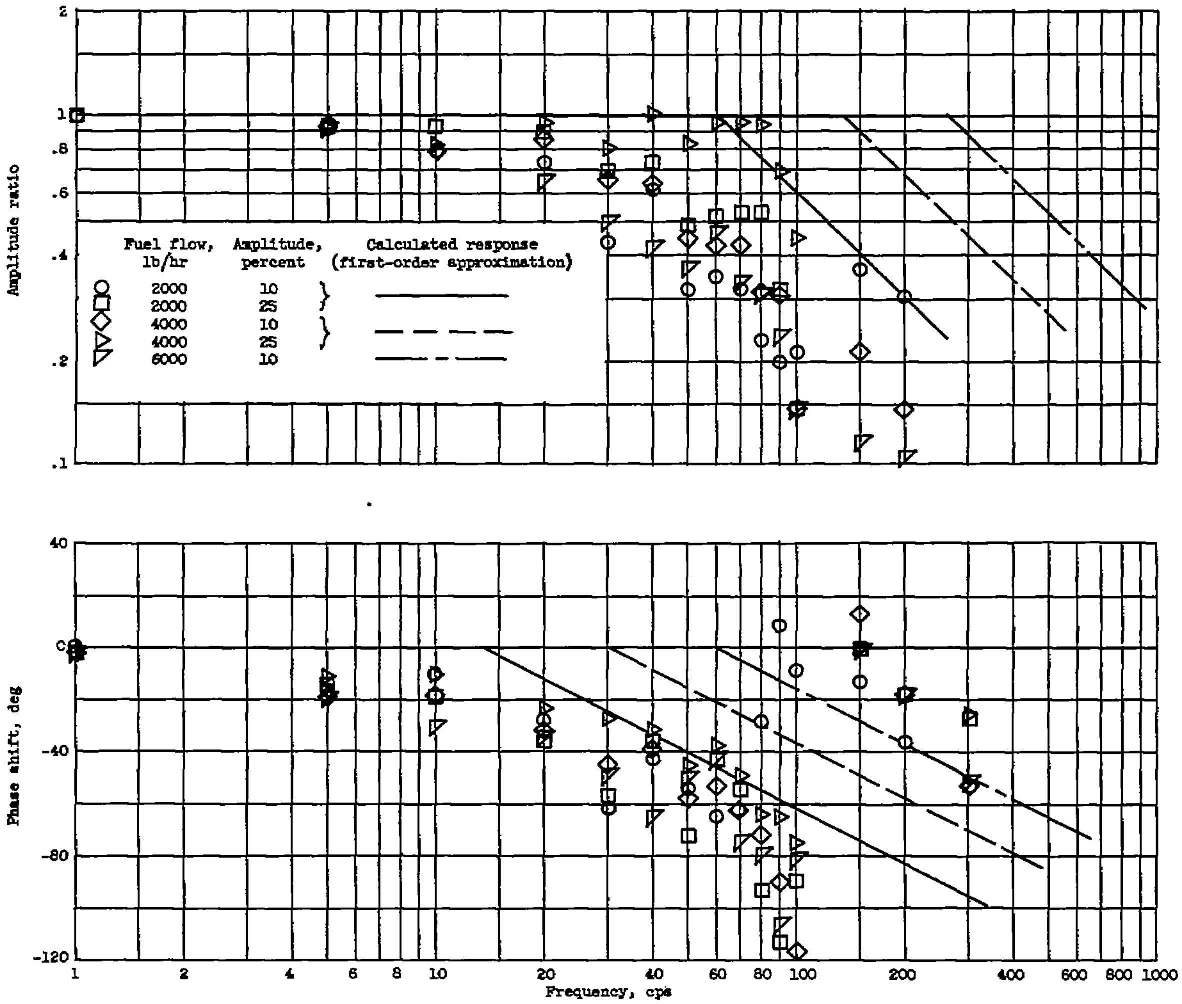
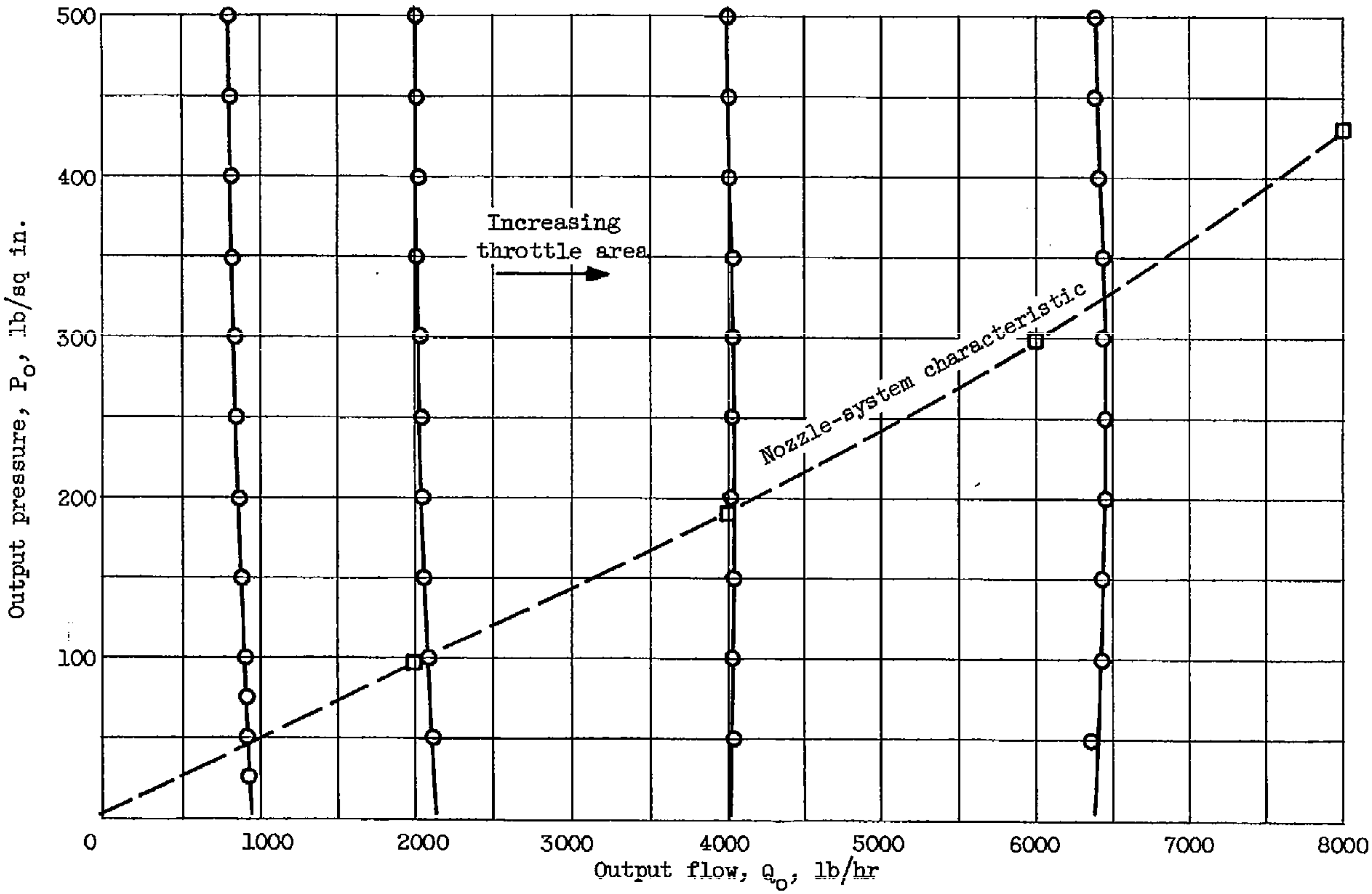



(b) Frequency response.

Figure 7. - Continued. Throttle plus bypass system.




(c) Output-impedance characteristics.

Figure 7. - Concluded. Throttle plus bypass system.

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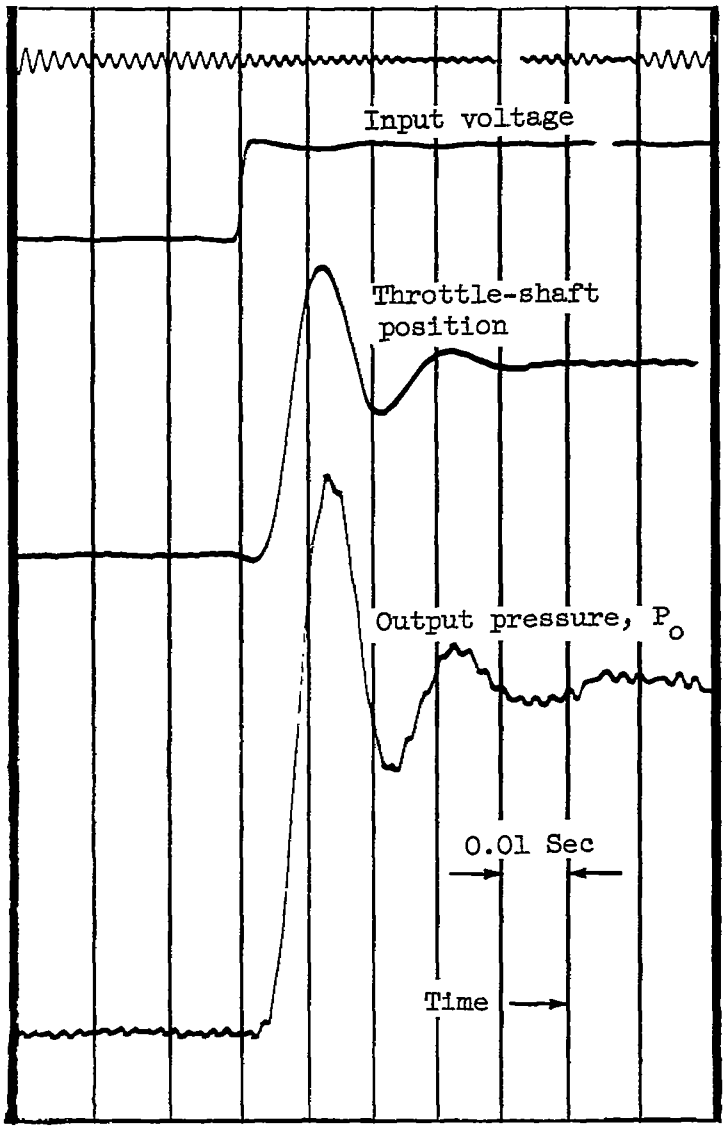
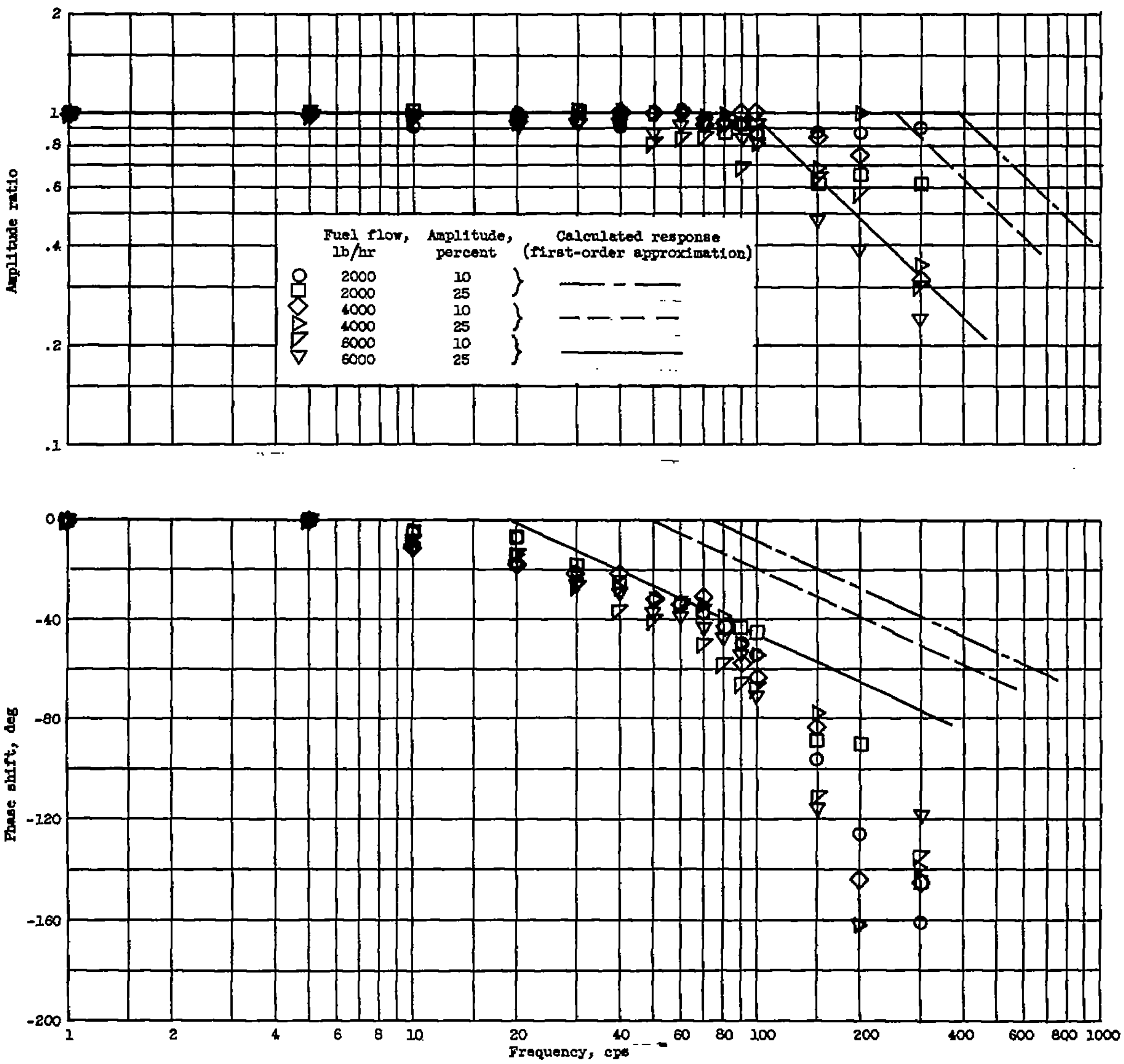


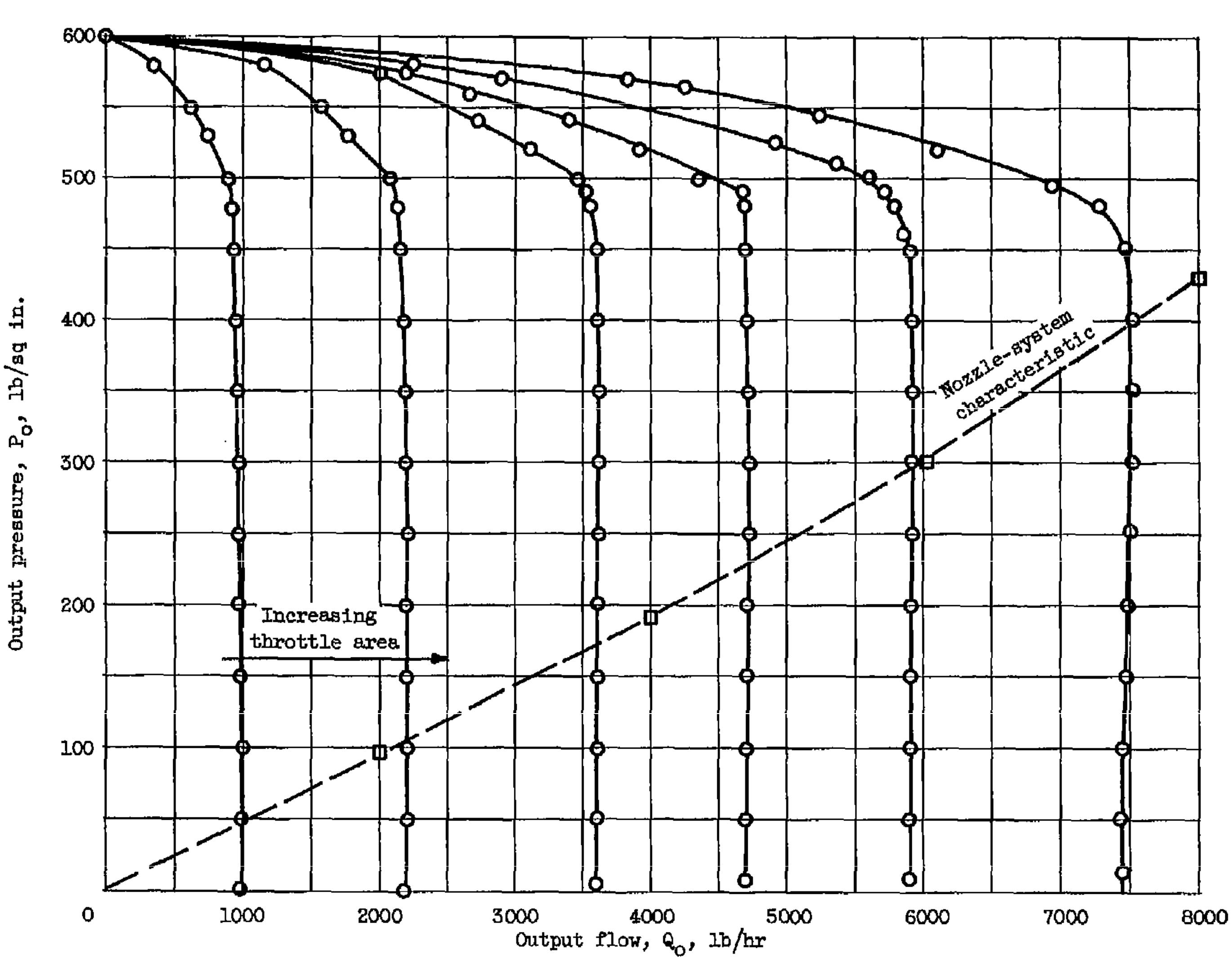

(a) Transient response. Initial fuel flow, 2000 pounds per hour; final fuel flow, 2500 pounds per hour.

Figure 8. - Throttle plus reducing-valve system.




(b) Frequency response.

Figure 8. - Continued. Throttle plus reducing-valve system.




(c) Output-impedance characteristics.

Figure 8. - Concluded. Throttle plus reducing-valve system.

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